Density and temperature of fermions and bosons from quantum fluctuations

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In recent years, the availability of heavy-ion accelerators which provide colliding nuclei from a few MeV/nucleon to GeV/nucleon and new and performing 4π detectors, has fueled a field of research loosely referred to as Nuclear Fragmentation. Fragmentation experiments could provide information about the nuclear matter properties and constrain the equation of state (EOS) of nuclear matter [1]. Even though a large variety of experimental data and refined microscopic models exist, to date it does not exist a method to determine densities and temperatures reached during the collisions, which takes into account the genuine quantum nature of the system. In this work we discuss some properties at finite temperatures assuming either a classical gas or a quantum system (Fermions or Bosons). We show that at the densities and temperatures of interest the classical approximation is not valid. We base our method on fluctuations estimated from an event-by-event determination of fragments arising from the energetic collision. We also include quadrupole fluctuations to have a direct measurement of densities and temperatures for subatomic systems for which it is difficult to obtain such informations in a direct way. We also suggest a method for calculating an excitation energy which should minimize collective effects and could be applied when a limited information is available, for example if only light cluster are measured. We apply the proposed method to microscopic CoMD approach [2] which includes fermionic statistics.

A method for measuring the temperature was proposed in [3] based on momentum fluctuations of detected particles. A quadrupole $Q_{xy} = \langle p_x^2 - p_y^2 \rangle$ is defined in a direction transverse to the beam axis (z-axis) to minimize non equilibrium effects and the average is performed, for a given particle type, over events. Such a quantity is zero in the center of mass of the equilibrated emitting source. Its variance is given by the simple formula:

$$\sigma_{xy}^2 = \int d^3p (p_x^2 - p_y^2)^2 f(p) \tag{1}$$

Where f(p) is the momentum distribution of particles. In [3] a classical Maxwell-Boltzmann distribution of particles at temperature T_{cl} was assumed which gives $\sigma_{xy}^2 = \overline{N} 4m^2 T_{cl}^2$, m is the mass of fragment. $\overline{N}$ is the average number of particles which could be conveniently normalized to one. In heavy ion collisions, the produced particles do not follow classical statistics thus the correct distribution function must be used in eq. (1). Protons(p), neutrons(n), tritium(t) ect. follow the Fermi-Dirac statistics while, deuterium(d), alpha(α), ect. should follow the Bose-Einstein statistics. In this work, we will concentrate on fermions and bosons respectively, in particular p and n for fermions and d and α for bosons which are abundantly produced in the collisions thus carrying important information on the densities and temperatures reached.

For fermions, we use a Fermi-Dirac distribution f(p) and expanding to $O(\frac{T}{\epsilon_f})^4$, where $\epsilon_f = \epsilon_{f0}(\frac{\rho}{\rho_0})^{\frac{2}{3}} = 36(\frac{\rho}{\rho_0})^{\frac{2}{3}}$ MeV is the Fermi energy of nuclear matter, we get [4]

$$\sigma_{xy}^2 = \overline{N}[\frac{16m^2\epsilon_f^2}{35}(1 + \frac{7}{6}\pi^2(\frac{T}{\epsilon_f})^2 + O(\frac{T}{\epsilon_f})^4)] \tag{2}$$

This result is in evident contrast with the classical one: even at zero T and ground density ρ_0, quadrupole fluctuations arise from the Fermi motion. The quadrupole fluctuations depend on termperature and density through ϵ_f, thus we need more information in order to be able to determine both quantities.

Within the same framework we can calculate the fluctuations of the p, n multiplicity distributions. These are given by [4]:

$$\frac{\langle(\Delta N)^2\rangle}{\bar{N}} = \frac{3}{2}\frac{T}{\epsilon_f} + O(\frac{T}{\epsilon_f})^3 \quad (3)$$

Substitute eq. (3) into eq. (2) givers the Fermi energy in terms of quadrupole and multiplicity fluctuations which can be measured in experiments. Knowing the Fermi energy we obtain the quantum temperature from eq. (3).

For bosons, we use a Bose-Einstein distribution f(p) for a particle of spin s, and expanding near the critical temperature $T_c = \frac{2\pi}{[2.612(2s+1)]^{2/3}}\frac{\hbar^2}{m}\rho^{2/3}$ at a given density ρ, we get [4]:

$$\sigma_{xy}^2 = \bar{N}(2mT)^2\frac{g_{\frac{7}{2}}(1)}{g_{\frac{3}{2}}(1)} \qquad (T < T_c) \quad (4)$$

$$\sigma_{xy}^2 = \bar{N}(2mT)^2\frac{g_{\frac{7}{2}}(z)}{g_{\frac{3}{2}}(z)} \qquad (T > T_c) \quad (5)$$

where the $g_n(z)$ functions are well studied in the literature [4] and $z = e^{\mu/T}$ is the fugacity which depends on the critical temperature for Bose condensation and thus on the density of the system and the chemical potential μ [4]. The quadrupole fluctuations depend on temperature and density through T_c, thus we need more information in order to be able to determine both quantities for $T > T_c$.

Within the same framework we can calculate the fluctuations of the d, α multiplicity distributions. These are given by [4]:

$$\frac{\langle(\Delta N)^2\rangle}{\bar{N}} = \left(\frac{T}{T_c}\right)^{3/2}[1 + \left(\frac{T}{T_c}\right)^{\frac{3}{2}}] \qquad (T < T_c) \quad (6)$$

$$\frac{\langle(\Delta N)^2\rangle}{\bar{N}} = 0.921\frac{\left(\frac{T}{T_c}\right)^3}{[1 - \left(\frac{T}{T_c}\right)^{\frac{3}{2}}]^2} \qquad (T > T_c) \quad (7)$$

Fluctuations are larger than the average value and might diverge near the critical point, eq. (7), in the indicated approximations. Interactions and finite size effects will of course smoothen the divergence [4]. These results are very important and could be used to pin down a Bose condensate.

Two solutions are possible depending whether the system is above or below the critical temperature for a Bose condensate. Below the critical point, eq. (4) can be used to calculate T and then eq. (6) gives the critical temperature and the corresponding density. Above the critical point it is better to estimate the chemical potential which, in the same approximation, is given by:

$$\frac{-\mu}{T} = \frac{1}{2}\frac{1}{\frac{\langle(\Delta N)^2\rangle}{\bar{N}}} \qquad (T > T_c) \quad (8)$$

Notice the similarity between eq. (3) and eq. (8), where the Fermi energy is substituted by the chemical potential. From this equation we can estimate the g_n functions entering eq. (5) and obtain the value of T. Using such a value in eq. (7), gives Tc and the density ρ.

To illustrate the strength of our approach we simulated $^{40}Ca+^{40}Ca$ heavy ion collisions at fixed impact parameter b=1 fm and beam energies E_{lab}/A ranging from 4 MeV/A up to 100 MeV/A. Collisions were followed up to a maximum time $t = 1000\ fm/c$ in order to accumulate enough statistics. Particles emitted at later times (evaporation) could affect somehow the results and this might be important especially at the lowest beam energies. The choice of central collisions was dictated by the desire to obtain full equilibration. This however, did not occur especially at the highest beam energies due to a partial transparency for some events. For this reason the quadrupole in the transverse direction, eq. (1), was chosen. Furthermore, in order to correct for collective effects as much as possible, we defined a 'thermal' energy as:

$$\langle \frac{E_{th}}{A} \rangle = \frac{E_{cm}}{A} - \left[\langle \frac{E_{object}}{N_{object} A_{object}} \rangle - \frac{3}{2} \langle \frac{E_{object\,xy}}{N_{object} A_{object}} \rangle \right] - Q_{value} \qquad (9)$$

where $\langle \frac{\mathbf{E_{object}}}{\overline{\mathbf{N_{object}}}\mathbf{A_{object}}} \rangle$ and $\langle \frac{\mathbf{E_{object\,xy}}}{\overline{\mathbf{N_{object}}}\mathbf{A_{object}}} \rangle$ are the average total and transverse kinetic energies per particle of object A_{object} , eg. A_{object} =1 for proton, A_{object} = 4 for α$\boldsymbol{Q}$. $\mathbf{Q_{value}} = \frac{\overline{\mathbf{N_{object}}}}{\mathbf{N_{object}}} \mathbf{8\ MeV}$. 8 MeV is the average binding energy of a nucleon, $\mathbf{N_{object}}$ the total objects of the system and $\overline{\mathbf{N_{object}}}$ the average number of objects emitted at each beam energy. For a completely equilibrated system, the transverse kinetic energy (times 3/2) is equal to the total kinetic energy and the term in the square bracket cancels.

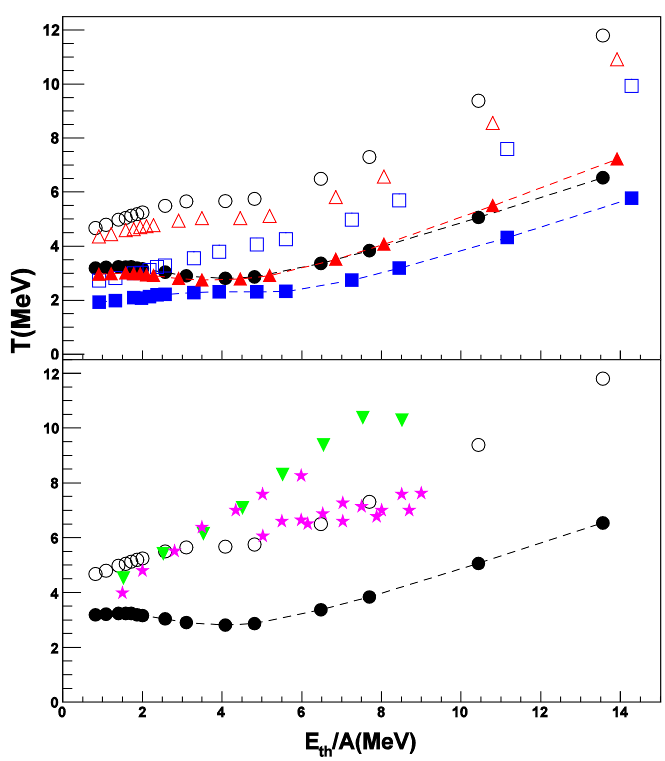

FIG. 1. Temperature versus thermal energy per particle derived from quantum fluctuations (full symbols joined by dashed lines) compared to the classical case (open symbols). (Top) Circles refer to protons, squares to neutrons and triangles to protons and neutrons. (Bottom) Same as above for protons. Data: down triangles from classical quadrupole fluctuations [3], star symbols from particle ratios [5].

All the center of mass energy, $\frac{E_{cm}}{A}$, is converted into thermal energy (plus the Q_{value}). In the opposite case, say an almost complete transparency of the collision, the transverse energy would be negligible and the resulting thermal energy would be small. Our approximation will account for some corrections, and this will become more and more exact when many fragment types are included in eq. (9) [3].

In fig. 1 (top) we plot the estimated temperatures for fermions at various 'thermal' energies both for the quantum (full symbols) and classical approximations (open symbols). As we see the quantum case is systematically lower than the classical one. We also notice a difference if the T are estimated from the proton distributions (circles) or neutrons (squares) or the sum of the two (triangles). This is clearly a Coulomb effect which gets smaller as expected at higher energies.

In fig. 2 we plot the ratio $\frac{T}{\epsilon_f}$ directly obtained from eq. (3), versus reduced density which is obtained from eqs. (2) and (3). The highest $\frac{T}{\epsilon_f}$ corresponds to the lowest beam energy as well and gives the lowest density, especially for the neutrons case. The top energy scale in the figure is for illustration purposes only and it refers to the neutron case. In fact at the same beam energy, p and pn might measure a different $\frac{T}{\epsilon_f}$ ratio respect to n. This result might be surprising at first, but it simply tells us that at the lowest energies nucleons from the surface of the colliding nuclei come into contact. Those nucleons are located in a low density region, especially neutrons which do not feel the Coulomb field. Thus this is the average density explored by the participant nucleons. With increasing beam energy, the overlapping region

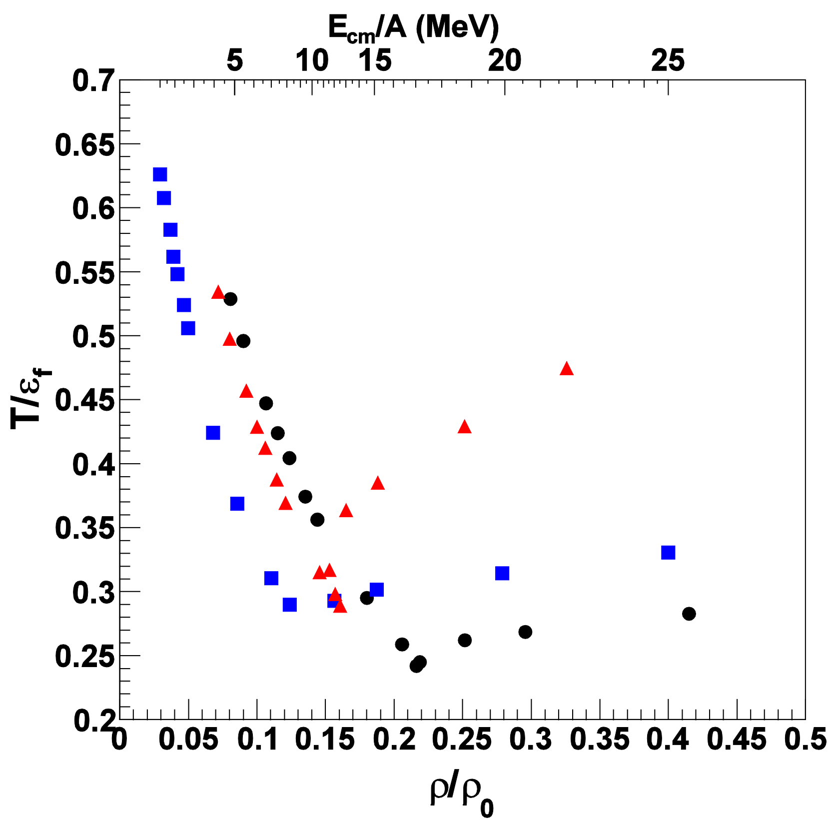

FIG. 2. Temperature divided the Fermi energy versus density normalized to the ground state one derived from quantum fluctuations, eqs. (1)-(3). Symbols as in Fig. 1. The top energy scale refers to the neutron case.

increases and more and more fermions are emitted. At about $\mathbf{E_{lab}/A \approx 20\,MeV/A}$ a large number of nucleons are excited and the emission from surface becomes a volume emission. It is important to stress that the ratio plotted in fig. 2 is always smaller than one which confirms the approximations used in eqs. (1)-(3).

In fig. 3 we plot the reduced variances versus excitation energy per particle. The Boson results are given by the full symbols, top panel. As we see in the figure, $\boldsymbol{\alpha}$ normalized fluctuations are generally larger than d-fluctuations. As we will show below, this implies that those particles might explore different regions of densities and temperatures. In both cases, fluctuations are large and, in some cases, above Poissonian for $\boldsymbol{\alpha' s}$. In order to understand if a Bose condensate occurs in the model (and in the future in experiments) it is instructive to compare the Boson normalized fluctuations to those of Fermions. In fig. 3 (bottom panel), normalized Fermion fluctuations are given. As we see the normalized fluctuations of p and n are much smaller than 1 at variance with the Boson case, which would suggest a condensate. However, heavier Fermion clusters such as $\mathbf{^3He}$ and tritons, display fluctuations larger than d and smaller than $\boldsymbol{\alpha}$. These facts are important to understand what is happening in the model and eventually search for an experimental confirmation. Notice in fig. 3 the occurrence of a minimum at a similar excitation energy for p, n and d but not for heavier clusters. This will have an effect on the EOS as we will show below.

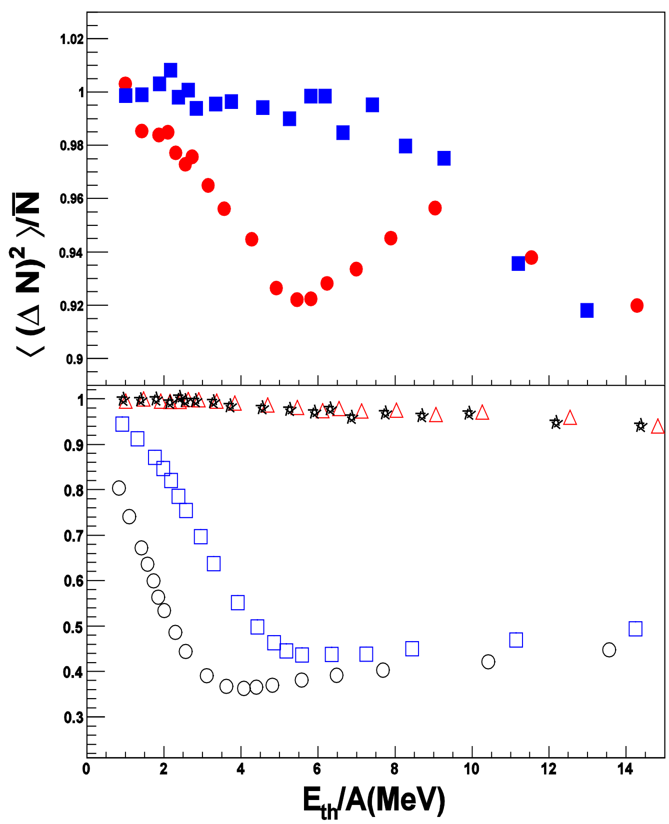

FIG. 3. Normalized variance versus excitation energy per nucleon. (Top panel) CoMD results for d (full circles) and α particles (full squares). For comparison the normalized fluctuations for fermions (bottom panel). (Open) Circles, squares and triangles refer to protons, neutrons and tritons, stars refer to ^{3}He. Notice the change of scales in the two panels.

It is interesting to discuss the densities 'seen' by the different Bosons during the reaction. A plot of density (divided by then ground state density) versus temperature (divided by the critical temperature for a condenstate) is given in fig. 4. Notice the peculiar behavior of d and **α** clusters. While the latter are formed at a constant reduced temperature but at different densities for each beam energy, the deuterons are formed always at a very small (constant) density but at different reduced temperatures. As we noticed above two effects are at play. The first is that there is no Pauli blocking for nucleons inside the clusters, the second is the different binding energies. Since d are over bound in the model (about 7 MeV) we expect that even smaller densities will be 'seen' in the data, the opposite we expect for **α**. These features remind of Mott transitions and in particular suggest that different particle types might be sensitive to different regions of the nuclear EOS as already noticed for Fermions [6, 7].

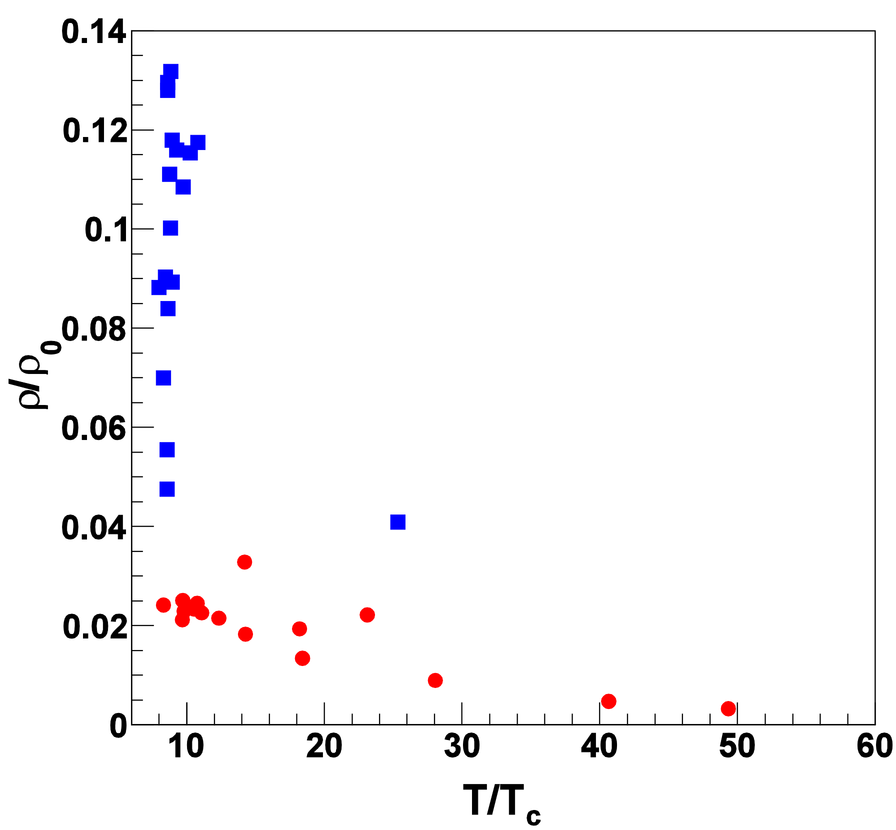

FIG. 4. Reduced density versus reduced temperature for Bosons. Symbols as in fig. 3.

In conclusion, we have addressed [6, 8] a general method for deriving densities and temperatures of fermions and bosons. In the framework of Constrained Molecular Dynamics model, which includes Fermi Statistics, we have discussed collisions of heavy ions below 100 MeV/A and obtained densities and temperatures at each bombarding energy. The results we have obtained here in a model case confirm that the classical approximation is unjustified. We have seen in this work that different particles like (p, n, d, α) explore different density and temperature regions. Open problems such as Mott transition, Bose condensate, pairing ect. in low density matter might be addressed through a detailed study of the EOS.

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